3. Measurement Theory

This section describes the theory and the mathematical basis for the measurement methods used for the services outlined in this publication. The measurement equation is developed and the generalizing assumptions are discussed. The measurement equation is then used in estimating the uncertainties in section 7. The calibration method using detector substitution is described first in general, and then, in detail for the measurement services.

3.1 Measurement Equation

Developing the measurement equation is fundamental to understanding the physics and optics involved with the measurement. This derivation provides an analytical foundation for the measurement process and the assumptions and approximations used in the measurement process. The measurement equation is also fundamental to the analysis of the uncertainty of the measurement process. In essence, a detector comparator is a spectroradiometer that measures the response of different detectors instead of different sources. Thus, the measurement equation developed for a general spectroradiometer can be adapted to detector spectral responsivity measurements.

The measurement equation presented here for spectral responsivity is developed following the general procedure described in chapter 5 of the Self-Study Manual on Optical Radiation Measurements [25] and is similar to the development of eq (7.18) in Ref. [26] for spectral irradiance when using a monochromator-based spectroradiometer. The measurement equation for spectral responsivity is

$$V(A, \Delta l, l_0) = \int_{\Delta l} \int_{A} E_l(x, y, l_0, l) \cdot S_F(x, y, l) \cdot \mathrm{d}A \cdot \mathrm{d}l \ \ [\mathrm{U}], \tag{3.1}$$

where $V(A, \Delta l, l_0)$ is the output signal (in U units, typically volts or amperes); $E_l(x, y, l_0, l)$ is the spectral irradiance function in l of the comparator system at detector position x,y for a wavelength setting of l_0; $S_F(x, y, l)$ is the spectral (radiant flux) responsivity of the detector; A is the area of the radiant flux beam at the detector (See fig. 3.1.); and Δl is the wavelength interval for which the value of E_l is not zero (i.e., the full-width bandpass). This equation is equivalent to eq (7.1) [26], with some minor changes in the notation, where A and Δl are left symbolically in V to indicate that V depends on how these elements are chosen.

To simplify the analysis, the responsivity S_F is assumed uniform throughout A, so,

$$S_F(x, y, l) = S_F(l) \ \ [\mathrm{U \cdot W^{-1}}]. \tag{3.2}$$

Thus, the responsivity S_F is no longer dependent on position (x,y) and can be removed from the area integral

$$V(A, \Delta l, l_0) = \int_{\Delta l} S_F(l) \cdot \int_{A} E_l(x, y, l_0, l) \cdot \mathrm{d}A \cdot \mathrm{d}l \ \ [\mathrm{U}]. \tag{3.3}$$

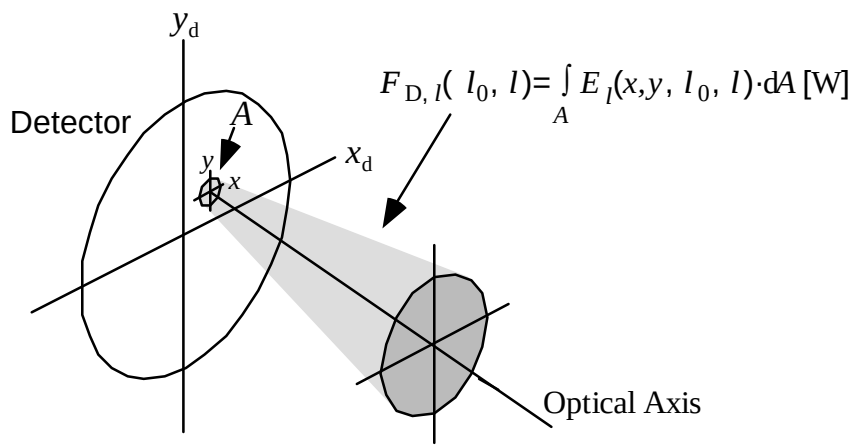

Figure 3.1. The geometry for detector spectral responsivity measurements.

Evaluating the area integral gives the spectral radiant flux function of the comparator system at the detector

$$F_{D,l}(l_0, l) = \int_A E_l(x, y, l_0, l)\cdot dA = t(l)\cdot F_l(l_0, l) \text{ [W]}, \tag{3.4}$$

where $F_l(l_0, l)$ is the output flux from the monochromator and $t(l)$ is the transmittance of the optics (and atmosphere) between the monochromator and the detector (See fig. 3.2.).

Considering only the spectral dependence of the signal for any given A, the measurement equation can be written

$$V(\Delta l, l_0) = \int_{\Delta l} S_F(l)\cdot t(l)\cdot F_l(l_0, l)\cdot dl \text{ [U]}. \tag{3.5}$$

Equation (3.5) is the flux equivalent to eq (7.1b) [26]. Introducing the slit-scattering function $z(l_0 - l)$ to the measurement equation allows the spectral radiant flux function of the monochromator $F_l(l_0, l)$ to be written as the product of two functions (with appropriate normalization)

$$F_l(l_0, l) = z(l_0 - l)\cdot F_{f,l}(l) \text{ [W]}, \tag{3.6}$$

where the slit-scattering function $z(l_0 - l)$ is dependent only on the difference between the wavelength setting of the monochromator and the wavelength of the flux and the "spectral flux" factor $F_{f,l}$ is dependent only on the wavelength of the flux. The factor $F_{f,l}$ is the spectral radiant flux at l_0, $F_l(l_0)$, and is equivalent to the responsivity factor r^f introduced in eq (7.12) [26]. Both the slit-scattering function $z(l_0 - l)$ and the factor $F_{f,l}$ can be determined experimentally, although the latter requires deconvolution from the measured output flux of the monochromator.

Thus the output signal V can now be written

$$V(\Delta l, l_0) = \int_{\Delta l} S_F(l) \cdot t(l) \cdot z(l_0 - l) \cdot F_{f,l}(l) \cdot \mathrm{d}l \text{ [U]}. \tag{3.7}$$

Equation (3.7) is the flux equivalent to eq (7.13) [26] and is in a form that can be used for finding the spectral responsivity of a detector. Thus a measurement equation has been developed for a spectrally selective source instead of a spectrally selective detector (spectroradiometer). If the product $S_F \cdot t \cdot F_{f,l}$ is approximately constant (or linear) over the spectral range for which $z(l_0 - l)$ is significant (i.e., within Δl) and z is approximately symmetrical with respect to l_0, eq (3.7) can be written

$$V(\Delta l, l_0) \cong S_F(l_0) \cdot t(l_0) \cdot F_{f,l}(l_0) \cdot \int_{\Delta l} z(l_0 - l) \times \mathrm{d}l \text{ [U]}. \tag{3.8}$$

Equation (3.8) is the equivalent flux version of eq (7.18) [26]. Implicit in this derivation is the assumption that the slit-scattering function $z(l_0 - l)$ does not change with the wavelength setting l_0 of the monochromator (that is, the dispersion is the same for all wavelengths) and thus $z(l_0 - l)$ need only be measured once. Optical aberrations, scattering, and diffraction are also assumed to be the same whether a monochromatic beam is varied in l over the monochromator bandpass, Δl, or l_0 of the monochromator is varied over a beam of fixed wavelength l.

Equation (3.7) is the measurement equation for an ideal monochromator. A real monochromator system has spectrally scattered light (also known as stray light or out-of-band radiation and hereafter will be referred to as simply stray light) due to imperfections in the monochromator (and other optics). This is light from outside the spectral region Δl which is scattered into Δl and which contributes to the measured signal. Adding a stray light term, $V_{sl}(\Delta l, l_0)$, to eq (3.8) gives

$$V(\Delta l, l_0) \cong S_F(l_0) \cdot t(l_0) \cdot F_{f,l}(l_0) \cdot \int_{\Delta l} z(l_0 - l) \cdot \mathrm{d}l + V_{sl}(\Delta l, l_0) \text{ [U]}, \tag{3.9}$$

where

$$V_{sl}(\Delta l, l_0) \cong \int_{l \neq \Delta l} S_F(l) \cdot z(l_0 - l) \cdot t(l) \cdot F_{f,l}(l) \cdot \mathrm{d}l \text{ [U]}. \tag{3.10}$$

The V_{sl} term in eq (3.9) is typically small for radiometric measurement systems and is normally ignored in the "routine" measurement equation but is included in the uncertainty estimate calculations in section 7.1.2. Equation (3.9) separates the measurement equation into two parts, the first term represents the in-band signal and the second term represents the out-of-band signal as indicated by the limits on each integral.

The integral remaining in the first term can be evaluated and combined with the "spectral flux" factor $F_{f,l}$ to give an expression for the flux leaving the monochromator F'_l,

$$F'_l(l_0) = F_{f,l}(l_0) \cdot \int_{\Delta l} z(l_0 - l) \cdot \mathrm{d}l \cong F_l(l_0, l) \text{ [W]}. \tag{3.11}$$

Using eq (3.11) the measurement equation (eq (3.9)) can now be written in a form that is easily applied to the situation of measuring detector spectral responsivity

$$V(\Delta l, l_0) \cong S_F(l_0) \cdot t(l_0) \cdot F'_l(l_0) \text{ [U]}. \tag{3.12}$$

Before developing a "routine" measurement equation, it is important to reiterate the simplifying assumptions that were made to develop eq (3.12).

3.1.1 Approximations

The assumptions made in the development of the measurement equation (eq (3.12)) are summarized and discussed below. The development of the measurement equation followed the procedure described in chapter 5 of the Self-Study Manual on Optical Radiation Measurements [25] and does not include parameters for time, polarization, or incident angle; nor does it include environmental parameters such as ambient temperature or humidity, corrections for diffraction effects (departure from geometrical (ray) optics), and nonlinear responsivity.

A key requirement to detector-based radiometry is that detector responsivities are stable over time. In general the light from any monochromator system can be polarized, thus the effect of polarization on the responsivity of the detectors needs to be evaluated. Generally, this is not a problem for most photodetectors since the detectors are measured at normal incidence to the optical axis (and the detector surfaces are isotropic). The effect of the converging beam angle on the reflectance (and transmittance) from the detector surface (and window) is small compared to the variance of repeated measurements and is typically neglected. However this is not the case when filters are used with the detectors (e.g., photometers), especially interference filters, where the transmission is a strong function of the angle of incidence. Also, the detector area must be larger than the optical beam so that all of the optical radiation is collected by the detector (i.e., the detectors are underfilled). This also requires that the detector Field-of-View (FOV) be larger than the optical beam from the comparator system. The detector size and FOV of the typical 1 cm^2 photodiodes NIST provides meet these requirements.

Humidity can affect the measurement by changing the transmittance t of the system and will be taken into account in the measurement equation developed below. Humidity can also affect the detector itself (or its window). For example, with windowless silicon photodiodes the absorption of water by the SiO_2 surface passivation layer changes the photodiode reflectivity which changes the responsivity [27]. Typically, for a windowed detector in the laboratory, effects due to water absorption (onto the detector or window) is not observed.

The temperature variation of the laboratory is small (typically < 1 °C) over the measurement time; therefore, the responsivity temperature dependence is neglected over most of the spectral region. When of concern, it can be applied as an additional uncertainty term.

Diffraction effects can be estimated from [28]

$$q^{\mathrm{d}} = 2.44 \cdot \frac{l}{d} \text{ [rad]}, \tag{3.13}$$

where q^{d} is the diffraction angle (for the first Airy disk), l is the longest wavelength of the system, and d is the diameter of the aperture at the monochromator exit slit (the smallest aperture in the system). For the Vis/NIR SCF $l \approx 2$ µm and $d \approx 1$ mm which gives

$$q^{\mathrm{d}}_{\mathrm{Vis/NIR}} = 2.44 \cdot \frac{2\ \mu\mathrm{m}}{1\ \mathrm{mm}} = 4.88 \times 10^{-3} \approx 5\ \mathrm{mrad}. \tag{3.14}$$

The Vis/NIR SCF has a focal ratio (f-number or $f/\#$) of ≈9 giving a beam angle of ≈110 mrad which is over 20 times greater than the diffraction limit. For the UV SCF (l ≈0.5 µm, d ≈1.5 mm, and $f/\#$ ≈5) the diffraction effects are even smaller due to the shorter wavelengths. In this case, the beam angle is ≈200 times greater than the diffraction limit. Thus diffraction effects can be ignored for most UV, visible, and near-IR systems of the type described in this publication. Likewise, coherence effects are negligible [29] and are ignored in the analysis of these comparator systems.

The responsivity S_F is assumed uniform throughout the area A of the incident beam on the detector. In practice, this is accomplished by reducing A (both mechanically with apertures and optically via imaging optics) so that S_F is uniform over A. This limits the amount of flux (power) that can be delivered to the detector but this does not hinder the responsivity measurements for typical photodiodes. Deviations from this approximation are considered as uncertainty terms.

Certain assumptions were made about the system [26] (primarily the monochromator). One assumption is that the dispersion remains the same for all wavelengths and that the slit-scattering function $z(l_0 - l)$ does not change with the wavelength setting l_0 of the monochromator. These approximations are not included in the uncertainties because their effect was determined to be negligible. This allows $z(l_0 - l)$ to be measured only once for an instrument and used over the entire spectral range. Another assumption is that optical aberrations, scattering, and diffraction are the same whether the l of a monochromatic source is varied over the monochromator bandpass Δl or whether the monochromator wavelength setting l_0 is varied over a monochromatic source of wavelength l. Also $z(l_0 - l)$ is assumed approximately symmetrical with respect to l_0 which is the case for the monochromators described in this publication.

The product $S_F \cdot t \cdot F_{\mathrm{f},l}$ is assumed to be constant (or linear) over Δl. In practice, this is affected by reducing Δl so that $S_F \cdot F_{\mathrm{f},l}$ is constant (or linear) over Δl. This also limits the amount of flux (power) that can be delivered to the detector, but this is not a hindrance for typical measurements. Most measurements are made on broadband detectors using a broadband source to provide a wide range of measurement wavelengths. However, this assumption may not be valid with some arc and discharge sources (primarily used in the UV) that can have strong spectral variations in output intensity. Typically, they are small and considered as uncertainty terms.

The spectrally scattered light is typically negligible for double monochromator systems but not for many single monochromator systems. Because the spectrally scattered light is small in magnitude it will not be used as a correction term in the measurement equation, but it will be evaluated and considered as an uncertainty term in section 7.

3.2 Substitution Method

The substitution method uses a standard detector to transfer its responsivity (output divided by input) to an unknown (test) detector. This method has a number of advantages that will be discussed in general and applied specifically to photodetectors. The measurement equation will then be developed and the general assumptions and their consequences discussed. Finally the measurement equation as applied to the systems described in this publication is developed.

3.2.1 General Substitution Method

The output (signal) Y from a detector (or system) with a linear response in U units, typically volts or amperes, can be written in general

$$Y_x = S_x \cdot X_x \text{ [U]}, \tag{3.15}$$

where Y_x is the signal from the detector, S_x is the responsivity of the detector, and X_x is the input to the detector. The signal from a standard detector with a known responsivity S_s is given by

$$Y_s = S_s \cdot X_s \text{ [U]}. \tag{3.16}$$

Dividing eq (3.15) by eq (3.16) we have

$$\frac{Y_x}{Y_s} = \frac{S_x \cdot X_x}{S_s \cdot X_s}. \tag{3.17}$$

If the input X to each system is assumed to be constant (this is the basis of the substitution method), solving for the detector responsivity S_x gives

$$S_x = \frac{Y_x}{Y_s} \cdot S_s \text{ [U·input units}^{-1}\text{]}. \tag{3.18}$$

Thus, using the substitution method, the standard detector responsivity S_s is scaled by the ratio of the outputs. The transfer is accomplished in essence by the standard detector measuring the input X to the test detector.

3.2.2 Photodetector Substitution

Photodetector responsivity measurements by detector substitution can be made using eq (3.18) and the following equipment: a broadband source, monochromator, focusing optics, and standard detector. (See fig. 3.2.) The radiant flux (power) F is the output flux from the source that enters the monochromator. The spectral radiant flux F_l is the output flux from the monochromator and t is the transmittance of any optics (and the atmosphere) between the monochromator and the detector. The spectral radiant flux received by the detectors is $F_{D,l} = t \cdot F_l$.

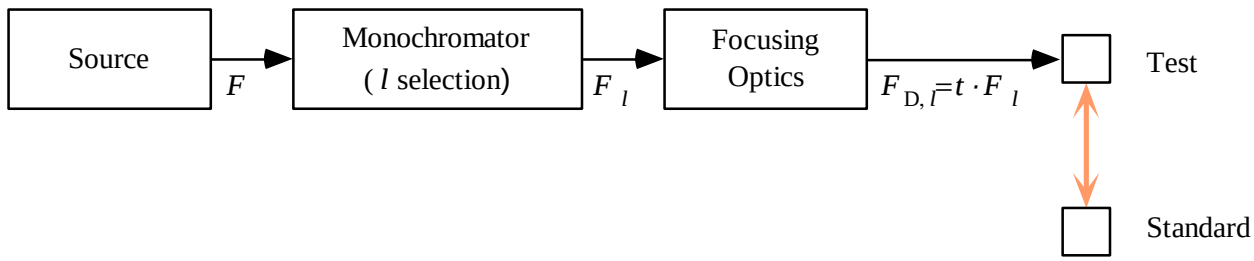

Figure 3.2. Block diagram of photodetector substitution method.

Using eq (3.18) assumes that the source is stable over the comparison time. This is generally true for incandescent sources (i.e., quartz-halogen (QTH) lamps), but less true for arc sources which are primarily used in the UV.

3.2.3 Substitution Method with Monitor

To eliminate the effect of power fluctuations, a beamsplitter and monitor detector are used. (See fig. 3.3.) Here $F_{D,l} = t_{bs} \cdot t \cdot F_l$ is the flux received by the detectors and $F_{M,l} = r_{bs} \cdot t \cdot F_l$ is the flux received by the monitor. The transmittance and reflectance of the beam splitter are t_{bs} and r_{bs}, respectively. The monitor detector and beamsplitter are assumed to be stable (constant) over the measurement time. Thus the monitor records the source power fluctuations and the ratio of detector to monitor signals will be constant.

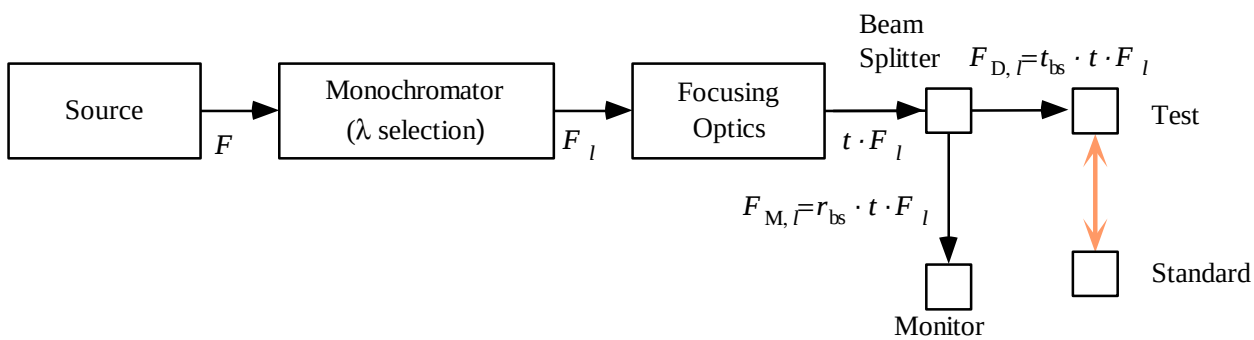

Figure 3.3. Block diagram of photodetector substitution method with monitor.

3.2.4 Measurement Equation Applied to the SCFs

The actual measurement equation is now developed. Dropping the l notation and changing the sub- and superscripts for clarity from eq (3.12), it can easily be shown that the signal from a detector x is

$$V_x = S_x \cdot G_x \cdot t_x \cdot F_x + V_{d,x} \text{ [V]}, \tag{3.19}$$

where S_x is the spectral responsivity in $A \cdot W^{-1}$, G_x is an explicit gain term for a transimpedance amplifier in $V \cdot A^{-1}$, t_x is the transmittance of any optics (and the atmosphere) between the monochromator and the detector, F_x is the output flux from the monochromator in W, and $V_{d,x}$ is the dark output (sometimes called the "background signal") in V, i.e., the signal produced when no flux is incident on the detector. (For photodiodes this is caused by their dark current.) In

practice $V_{d,x}$ is found by measuring the signal from the detector with the shutter closed at the exit slit of the monochromator. The net signal ΔV_x is the signal due to the radiant (optical) flux received by the detector

$$\Delta V_x = V_x - V_{d,x} = S_x \cdot G_x \cdot t_x \cdot F_x \text{ [V]}. \tag{3.20}$$

This is the "routine" measurement equation for the substitution method depicted in figure 3.2. The measurement equation for the "substitution method with monitor" (fig. 3.3) is written by explicitly including the transmittance t_{bs} and reflectance r_{bs} of the beam splitter assembly (which also contains the reflectance of the monitor turning mirror). The systems described in this publication simultaneously sample both the detector and monitor. The signals from the test detector x and monitor detector mx are

$$\Delta V_x = V_x - V_{d,x} = S_x \cdot G_x \cdot t_{bsx} \cdot t_x \cdot F_x \text{ [U]}, \tag{3.21}$$

and

$$\Delta V_{mx} = V_{mx} - V_{d,mx} = S_{mx} \cdot G_{mx} \cdot r_{bsx} \cdot t_x \cdot F_x \text{ [U]}. \tag{3.22}$$

The ratio of these two signals is

$$R_x = \frac{\Delta V_x}{\Delta V_{mx}} = \frac{S_x \cdot G_x \cdot t_{bsx} \cdot t_x \cdot F_x}{S_{mx} \cdot G_{mx} \cdot r_{bsx} \cdot t_x \cdot F_x}, \tag{3.23}$$

where $t_{bsx} \cdot t_x \cdot F_x$ is the fraction of light received by the test detector, and $r_{bsx} \cdot t_x \cdot F_x$ is the fraction of light received by the monitor.

There is a similar ratio for the two signals from the working standard detector and monitor,

$$R_s = \frac{\Delta V_s}{\Delta V_{ms}} = \frac{S_s \cdot G_s \cdot t_{bss} \cdot t_s \cdot F_s}{S_{ms} \cdot G_{ms} \cdot r_{bss} \cdot t_s \cdot F_s}. \tag{3.24}$$

Dividing eq (3.23) by eq (3.24) (taking the ratio of the ratios) gives

$$\frac{R_x}{R_s} = \frac{S_x \cdot G_x \cdot t_{bsx} \cdot t_x \cdot F_x}{S_{mx} \cdot G_{mx} \cdot r_{bsx} \cdot t_x \cdot F_x} \cdot \frac{S_{ms} \cdot G_{ms} \cdot r_{bss} \cdot t_s \cdot F_s}{S_s \cdot G_s \cdot t_{bss} \cdot t_s \cdot F_s}. \tag{3.25}$$

Looking at eq (3.25), it is now seen that variations or drifts in the source flux or system transmittance during the time between the measurement of the test detector and the working standard detector are canceled by the monitor detector. If the beamsplitter and monitor detector responsivity and amplifier gain are stable (constant) over the comparison time, other terms cancel, leaving

$$\frac{R_x}{R_s} = \frac{S_x \cdot G_x}{S_s \cdot G_s}. \tag{3.26}$$

Solving eq (3.26) for the test detector spectral responsivity S_x and substituting the signal measurements into the ratios we have

$$S_{\mathrm{x}} = \frac{R_{\mathrm{x}}}{R_{\mathrm{s}}} \cdot \frac{G_{\mathrm{s}}}{G_{\mathrm{x}}} \cdot S_{\mathrm{s}} = \frac{\dfrac{V_{\mathrm{x}} - V_{\mathrm{d,x}}}{V_{\mathrm{mx}} - V_{\mathrm{d,mx}}}}{\dfrac{V_{\mathrm{s}} - V_{\mathrm{d,s}}}{V_{\mathrm{ms}} - V_{\mathrm{d,ms}}}} \cdot \frac{G_{\mathrm{s}}}{G_{\mathrm{x}}} \cdot S_{\mathrm{s}} \ [\mathrm{A \cdot W^{-1}}]. \tag{3.27}$$

This is the working form of the measurement equation. Note that it still has the general form

$$S_{\mathrm{x}} = \frac{Y_{\mathrm{x}}}{Y_{\mathrm{s}}} \cdot S_{\mathrm{s}} \ [\mathrm{U \cdot input\ units^{-1}}]. \tag{3.18}$$

Depending on how the monochromator wavelength scale is calibrated, a correction term may be needed when the responsivity curves of the test and working standard detectors have different slopes. If the centroid wavelength of the bandpass is used to calibrate the monochromator wavelength scale, then there is no correction term. But if the peak wavelength of the bandpass is used, then eq (3.27) requires a correction term. Thus the measurement equation becomes

$$S_{\mathrm{x}} = \frac{R_{\mathrm{x}}}{R_{\mathrm{s}}} \cdot \frac{G_{\mathrm{s}}}{G_{\mathrm{x}}} \cdot S_{\mathrm{s}} + C_{\mathrm{bw}} \ [\mathrm{A \cdot W^{-1}}], \tag{3.28}$$

where C_{bw} is a correction term due to the bandpass of the monochromator and is referred to as the "bandwidth-effect." Like the stray light term, the bandwidth-effect is small for this system and is ignored in the "routine" measurement equation. The bandwidth-effect is analyzed in section 7.1.2 as an uncertainty term and included in the uncertainty estimate.

4. Equipment Description

In this section, the Visible to Near-Infrared Spectral Comparator Facility (Vis/NIR SCF) and Ultraviolet Spectral Comparator Facility (UV SCF) components are described along with the associated electronics.

4.1 Visible to Near-Infrared (Vis/NIR) Comparator Description

The Visible to Near-Infrared Spectral Comparator Facility (Vis/NIR SCF) is a monochromator-based system that typically measures the uniformity and absolute spectral responsivity of photodiodes in the 350 nm to 1800 nm spectral region. The Vis/NIR SCF operates from 350 nm to 1100 nm using silicon photodiodes as working standards and from 700 nm to 1800 nm using germanium photodiodes as working standards.

The Vis/NIR SCF uses the direct substitution method and automated translation stages to position the photodetectors for measurement. The test detectors as well as the working standards are fixed onto optical mounts that rotate and tilt for accurate alignment. A variety of sources can be selected. Typically a 100 W quartz-halogen lamp is used as the source in the Vis/NIR SCF. A shutter is located just after the monochromator exit slit. A monitor detector located after the monochromator measures source fluctuations. The detectors and the exit optics are enclosed in a